Corrected contact dynamics for the Steinecke and Herzel asymmetric two-mass model of the vocal folds

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Abstract: The simplified two-mass model of human vocal folds, proposed by Steinecke and Herzel [J. Acoust. Soc. Am. **97**(3), 1874–1884 (1995)], has seen widespread use throughout the speech community. Herein, an error is corrected in the contact loadings on colliding vocal folds with asymmetric tissue properties, as arises clinically in cases of unilateral paralysis. A revised contact model is proposed that remediates the erroneous asymmetric contact forces. The vibration regime map presented in Steinecke and Herzel illustrating the dynamical behavior of the system is revised using the corrected collision model.

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1. Introduction

Human speech is a complex phenomenon involving coupled fluid, structure, and acoustic interactions in the vocal tract.[1] Energy is exchanged between the air flowing through the vocal folds and the viscoelastic vocal tissue. Originally proposed by Ishizaka and Flanagan,[2] two-element lumped-mass models are often used to illustrate the causal coupled fluid-structure interactions of human speech. Two mass models represent each vocal fold as two coupled oscillating masses and the associated tissue properties with springs and dampers. By using two masses to model each vocal fold the primary oscillation modes are captured, namely the bulk displacement and the mucosal wave. The Ishizaka and Flanagan model assumes bilateral symmetry and non-linear springs to model tissue properties. A simplified two-mass model proposed by Steinecke and Herzel[3] uses linear springs and introduces asymmetric tissue properties. The

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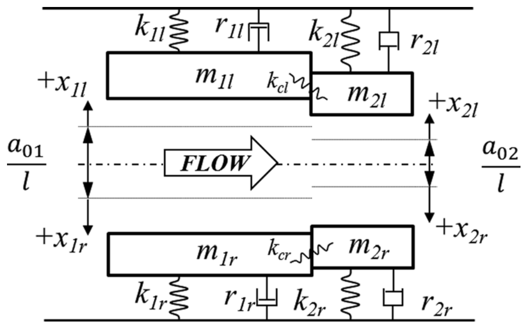

Fig. 1. Schematic representation of the simplified two-mass model proposed by Steinecke and Herzel (Ref. 3).

Steinecke and Herzel model, herein referred to as the "SH95 model," has seen widespread implementation in the study of various vocal pathologies and speech phenomena. A schematic overview of the SH95 model is shown in Fig. 1, adopting the nomenclature established in the original publication.[3]

Figure 1 shows a cross section of the vocal fold model in the coronal plane; the model is extruded a distance l into the page. The equation of motion for each of the four masses depicted in Fig. 1 is given by

$$m_{i\alpha}\ddot{x}_{i\alpha} + r_{i\alpha}\dot{x}_{i\alpha} + k_{i\alpha}x_{i\alpha} + \Theta(-a_i)c_{i\alpha}(a_i/2l) + k_{c\alpha}(x_{i\alpha} - x_{j\alpha}) = F_i, \tag{1}$$

where m is mass, x is displacement of the mass, r is the damping coefficient, and k and c are spring constants, the latter not indicated on Fig. 1 as it arises only during collision. The subscript i denotes the inferior or superior mass, 1 or 2, the subscript j has the opposite value of i, that is, 2 or 1, and the subscript α refers to the left or right side, l or r. The subscript c denotes that the associated spring is the coupling spring between the two masses of a given vocal fold. Aerodynamic loading is applied to each mass by F_i and the function Θ is defined such that $\Theta(x) = 1$ for $x > 0$ and $\Theta(x) = 0$ otherwise. Thus, Θ dictates when the contact forces are included in the dynamics.[3]

Contact between contralateral masses is modeled by the addition of two springs that are activated when the masses overlap, i.e., the area between opposing masses a_i is less than zero. The contact force experienced by the colliding masses is proportional to the deformation of the contact springs, which, in the SH95 model, is calculated from the displacement and coordinate offset of each mass

$$a_{ir} = a_{0i}/2 + lx_{ir}, \tag{2}$$

$$a_{il} = a_{0i}/2 + lx_{il}, \tag{3}$$

where a_{0i} is the area between the masses when they are in their rest positions. The total upper and lower areas are found by summing the left and right contributions

$$a_i = a_{ir} + a_{il}. \tag{4}$$

When adjacent upper or lower masses collide, the calculated area of the channel will be negative and the $\Theta(-a_i)c_{i\alpha}(a_i/2l)$ term in Eq. (1), which determines the contact force $F_{C,i\alpha}$ on each mass, is activated.

The SH95 model introduces an asymmetry parameter, Q, allowing asymmetric tissue properties as necessary for modeling pathologies such as unilateral paralysis. As discussed in Sec. 2, it is this asymmetry parameter that leads to non-physical manifestations of the contact forces that require correction.

2. Erroneous calculation of asymmetric contact forces

The asymmetry parameter, Q, in the SH95 model results in asymmetric contact spring stiffnesses as $c_{ir} = Qc_{il}$ to model unilateral superior nerve paralysis, and $c_{1r} = Qc_{1l}$ for recurrent laryngeal nerve paralysis, assuming the right vocal fold is the affected fold. When modeling unilateral paralysis using the SH95 model, it is noted that an error arises in the manner in which the contact forces between colliding left and right masses are calculated. Specifically, the error pertains to the development of asymmetric contact forces between the left and right vocal folds. By Newton's third law of motion, two objects colliding will experience an equal and opposite force; the manner in which the SH95 model calculates contact forces does not adhere to this and is thus incorrect for all cases with asymmetric tissue parameters. This error is alluded to by Dresel *et al.*,[4] although to the best of the authors' knowledge there has yet to be an explicit correction put forth to be applied to current implementations of the SH95 model.

Referring to Eq. (1), the governing equation of the SH95 dynamical model, it is noted that the contact force on each mass is calculated from the contact spring stiffness $c_{i\alpha}$ and the average overlap area distance $a_i/2l$ of the two contralateral masses. Calculating the contact forces based on the average overlap area is indeed valid for the symmetric implementation, as originally proposed by Ishizaka and Flanagan.[2] With symmetric tissue properties, the contact spring constants are always equal and thus the displacements of both springs should be equal, as shown in Fig. 2(a). However, assuming equal spring displacements when the spring stiffnesses are not equal, as in the case of asymmetric tissue properties, leads to unequal contact forces on the two opposing masses, a clear violation of Newton's third law. Thus, by assuming the deformations remain symmetric, the SH95 dynamical model over-estimates the contact forces on the healthy vocal fold and under-estimates the contact forces on the pathological vocal fold. Thus, the spring deformations should be dependent on Q such that the contact forces are equal on the two opposing vocal folds, as shown in Fig. 2(b).

3. Development of revised contact model

To correctly calculate the contact forces when modeling asymmetric tissue properties, a dynamical model is formulated by noting that the forces on the two colliding bodies will remain equal throughout the collision. For the forces to remain equal, the contact springs must deform such that the deformation of the two springs $\delta_{i\alpha}$ are related by the asymmetry parameter Q, as

$$\delta_{ir} = \delta_{il}/Q. \tag{5}$$

The total deformation of the colliding masses is calculated as $x_{ir} + x_{il} - a_{0i}/l$, which is the distance between the masses minus the offset of their local coordinate axes. Furthermore, it must hold that the sum of the deformations of the left and right contact spring are equal to the total deformation, as

$$\delta_{il} + \delta_{ir} = x_{ir} + x_{il} - a_{0i}/l. \tag{6}$$

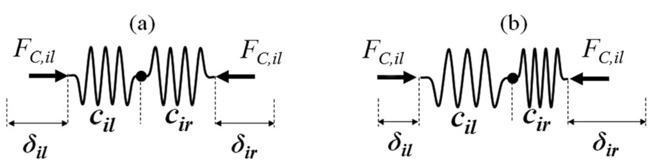

Fig. 2. (a) Symmetric case: both springs are of equal stiffness $c_{ir} = c_{il}$ and the springs undergo equal deformation $\delta_{ir} = \delta_{il}$ with equal and opposite contact forces $F_{C,i\alpha}$ developing. (b) Asymmetric case: the stiffness of the contact springs are related by the asymmetry parameter such that $c_{ir} = Qc_{il}$ and it follows that the deformations must also be related such that $\delta_{ir} = \delta_{il}/Q$ in order for the contact force to remain symmetric.

Solving the linear system formed from Eqs. (5) and (6), expressions for the individual deformations of each contact spring are found; the deformation of the right, that is the affected vocal fold, is given by Eq. (7) and the deformations of the left (healthy) vocal fold by Eq. (8)

$$\delta_{ir} = [1/(Q+1)](x_{il} + x_{ir} - a_{0i}/l), \tag{7}$$

$$\delta_{il} = [Q/(Q+1)](x_{il} + x_{ir} - a_{0i}/l). \tag{8}$$

Thus, the original governing differential equation, presented as Eq. (1), should be modified to account for the asymmetric contact forces. The revised dynamical model is as follows for right and left masses, respectively, assuming the right vocal fold is the affected fold:

$$m_{ir}\ddot{x}_{ir} + r_{ir}\dot{x}_{ir} + k_{ir}x_{ir} + \Theta(-a_i)c_{ir}(1/(Q+1))(x_{il} + x_{ir} - a_{0i}/l) + k_{cr}(x_{ir} - x_{jr}) = F_i, \tag{9}$$

$$m_{il}\ddot{x}_{il} + r_{il}\dot{x}_{il} + k_{il}x_{il} + \Theta(-a_i)c_{il}(Q/(Q+1))(x_{il} + x_{ir} - a_{0i}/l) + k_{cl}(x_{il} - x_{jl}) = F_i. \tag{10}$$

Modified differential equations of the form of Eqs. (9) and (10) are used to easily augment existing implementations of the SH95 model to correct the otherwise non-physical contact dynamics.

4. Effect of revised contact dynamics

Having identified a simple modification to the SH95 model to remediate the calculation of contact forces, it remains to evaluate the effect of the revision on the results of simulations. The effect of the revision is readily observed by tracking the magnitude of the contact force acting on each mass element through the duration of a collision, as shown in Fig. 3 for a subglottal pressure of $P_s = 1.45\,\text{kPa}$ and an asymmetry factor of $Q = 0.53$. Figure 3(a) shows the contact forces calculated with the original SH95 model; the lower masses collide followed by the upper masses, showing the characteristic mucosal wave. The magnitudes of the left and right contact forces are not equal for the upper or lower masses throughout the collision. Results from the revised contact model, as prescribed above, are shown in Fig. 3(b), where the magnitude of left and

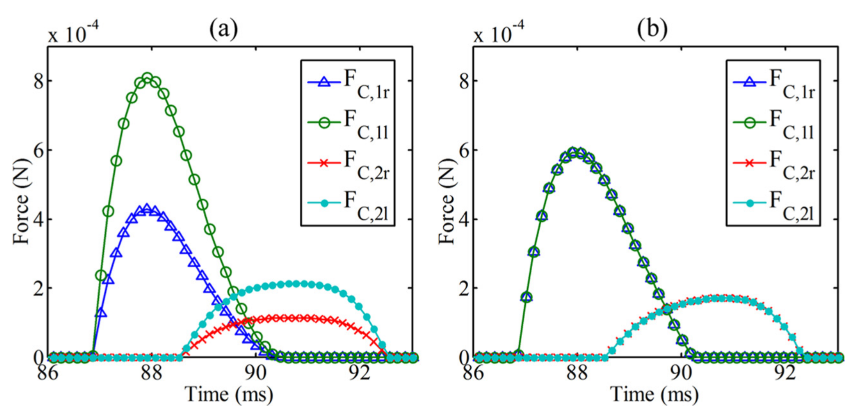

Fig. 3. (Color online) Comparison of contact forces for $P_s = 1.45\,\text{kPa}$ and $Q = 0.53$ through one complete closing cycle. (a) Original SH95 model, and (b) revised contact dynamics.

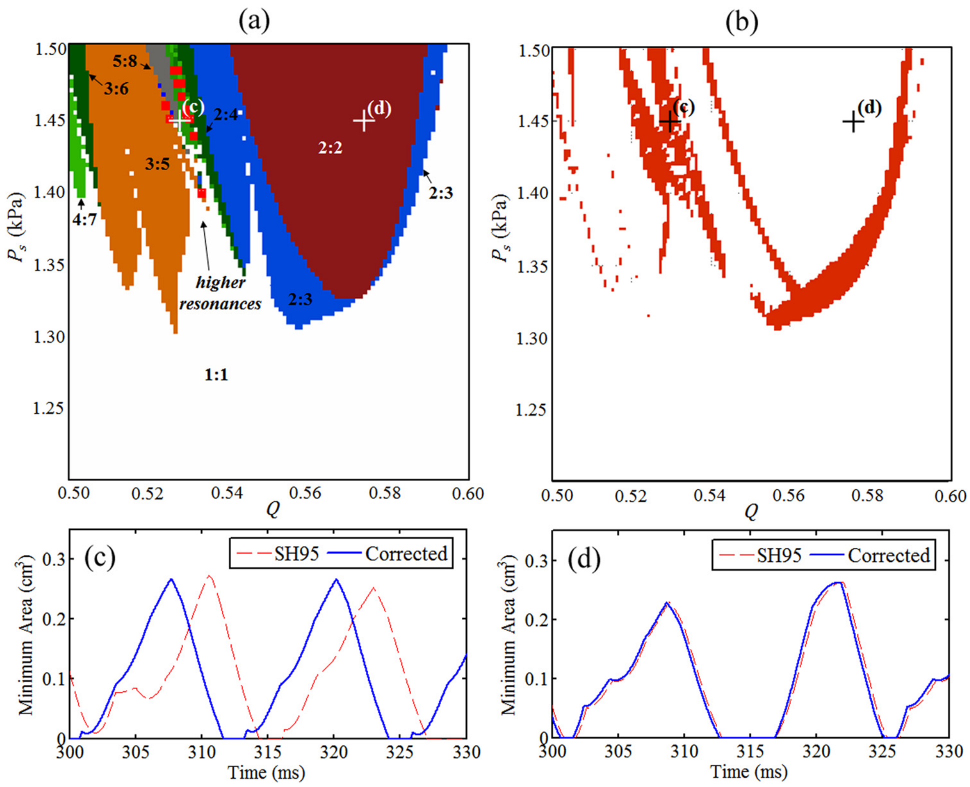

Fig. 4. (Color online) Comparison of the revised contact model and the original SH95 model. (a) Q-P_s plane regime plot generated with the corrected dynamical equations, (b) binary difference plot showing any regions that vary from the original SH95 model regime map (Ref. 3). Minimum glottal area calculated with the corrected and original SH95 models at (c) $P_s = 1.45$ kPa, $Q = 0.53$ and (d) $P_s = 1.45$ kPa, $Q = 0.57$.

right contact forces are equal for both the upper and lower masses. Contact forces have changed such that the right side experiences an increased contact force and the left a reduced contact force with the magnitude of the difference being inversely proportional to the asymmetry parameter Q.

A more holistic evaluation of the influence of the revision is gained through discrete sampling from the Q-P_s plane. A two-parameter bifurcation diagram is generated with the corrected SH95 model and presented in Fig. 4(a), while the regions that differ from the original SH95 regime map are highlighted in Fig. 4(b). The revised contact dynamics alter the vibration regime in the Q-P_s zones identified in SH95 to be associated with higher resonances, long transients, and chaos [see Fig. 4(b)]; the significance of chaos and higher order dynamics in voice production is known[5] and the effect of the corrected contact dynamics may be of significance to those making use of the SH95 model. In addition to influencing the dynamics, the revised model can alter important acoustic measures of speech, such as radiated sound pressure level, spectral tilt, etc. For Q-P_s combinations that result in a different oscillation ratio between the original SH95 and revised models, the effect of the contact fix on the acoustics is quite pronounced, as exemplified by the minimum area history given in Fig. 4(c). For regions that the oscillation ratio remains unaffected by the revision, the effect seems less prominent, as for the case shown in Fig. 4(d). The changes of minimum area history in these regions consist primarily of a phase shift, with slight changes in magnitude, while the signals remain similar in frequency.

5. Conclusions

Herein we presented a correction to an error in the contact force formulation in the original Steinecke and Herzel[3] two-mass vocal fold model when the tissue properties are asymmetric. The corrected equations of motion should be used in place of those presented by Steinecke and Herzel. We have shown that employing the corrected contact force model causes potentially significant changes in the dynamics for a given subglottal pressure and asymmetry factor. The recent developments of asymmetric fluid loading models[6] and interest in utilizing chaos as a potential diagnostic technique[7] make it critical that the governing equations of the simplified model be correct.

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