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¿Aporte para Agustín?

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Sea y una función continua de t.
Obedeciendo a Brook Taylor (1685-1731) y Colin Maclaurin (1698-1746):

$$y(t)\Big|_{t=(k+1)\cdot\Delta t} = y(t)\Big|_{t=k\cdot\Delta t} + \left.\frac{dy(t)}{dt}\right|_{t=k\cdot\Delta t}\cdot(\Delta t) + \frac{1}{2}\cdot\left.\frac{d^2y(t)}{dt^2}\right|_{t=k\cdot\Delta t}\cdot(\Delta t)^2 + \ldots\frac{1}{m!}\cdot\left.\frac{d^my(t)}{dt^m}\right|_{t=k\cdot\Delta t}\cdot(\Delta t)^m + \ldots \tag{1}$$

Si:

$$y(t) = y(0) + \int_0^t x(\tau)\cdot d\tau \tag{2}$$

entonces:

$$\begin{aligned} &\frac{dy(t)}{dt} = x(t) \\ &\frac{d^2y(t)}{dt^2} = \frac{dx(t)}{dt} \\ &\ldots \\ &\frac{d^my(t)}{dt^m} = \frac{d^{m-1}x(t)}{dt^{m-1}} \\ &\ldots \end{aligned} \tag{3}$$

e:

$$\begin{aligned} y(t)\Big|_{t=(k+1)\cdot\Delta t} &= y(t)\Big|_{t=k\cdot\Delta t} \\ &+ x(t)\Big|_{t=k\cdot\Delta t}\cdot(\Delta t) + \frac{1}{2}\cdot\left.\frac{dx(t)}{dt}\right|_{t=k\cdot\Delta t}\cdot(\Delta t)^2 + \ldots\frac{1}{m!}\cdot\left.\frac{d^{m-1}x(t)}{dt^{m-1}}\right|_{t=k\cdot\Delta t}\cdot(\Delta t)^m + \ldots \end{aligned} \tag{4}$$

Si no se dispone de $dx(t)/dt$, $dx^2(t)/dt^2$,... $dx^{m-1}(t)/dt^{m-1}$,... , se puede usar aproximaciones de Leonhard Paul Euler (1707-1783):

$$\frac{dx(t)}{dt} \approx \frac{x(t)\big|_{t=k\cdot\Delta t} - x(t)\big|_{t=(k-1)\cdot\Delta t}}{\Delta t}$$

$$\frac{d^2x(t)}{dt^2} \approx \frac{x(t)\big|_{t=k\cdot\Delta t} - 2\cdot x(t)\big|_{t=(k-1)\cdot\Delta t} + x(t)\big|_{t=(k-2)\cdot\Delta t}}{(\Delta t)^2} \tag{5}$$

...

$$\frac{d^{m-1}x(t)}{dt^{m-1}} = \frac{\sum_{n=0}^{m-1}(-1)^n \cdot \frac{(m-1)!}{n!\cdot(m-1)!} \cdot x(t)\big|_{t=(k-n)\cdot\Delta t}}{(\Delta t)^{m-1}}$$

...

con lo cual:

$$y(t)\big|_{t=(k+1)\cdot\Delta t} \approx y(t)\big|_{t=k\cdot\Delta t} + \left(x(t)\big|_{t=k\cdot\Delta t} + \frac{1}{2}\cdot\left(x(t)\big|_{t=k\cdot\Delta t} - x(t)\big|_{t=(k-1)\cdot\Delta t} \right) + \dots \frac{1}{m}\cdot\sum_{n=0}^{m-1}(-1)^n \cdot \frac{1}{n!\cdot(m-1)!}\cdot x(t)\big|_{t=(k-n)\cdot\Delta t} + \dots \right)\cdot(\Delta t) \tag{6}$$