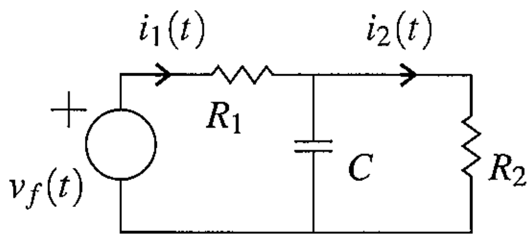
Solución

ELO102 – S1 2013 – Control #15 – 30 de agosto de 2013

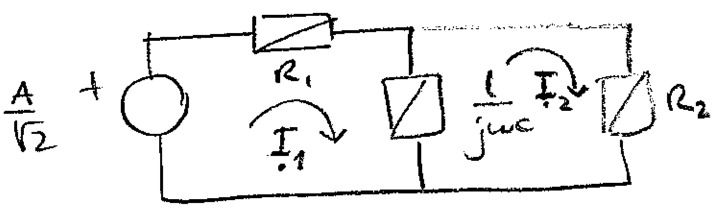
Problema 15.1 *En la red de la figura,* $v_f(t) = A\cos(\omega t)$

(a) Determine la corriente $i_2(t)$ *en estado estacionario.*

(b) Determine la potencia activa entregada por la fuente de tensión.



Aplicando mallas en el dominio de la T.F.:



$$\begin{bmatrix} R_1 + \frac{1}{j\omega C} & -\frac{1}{j\omega C} \\ \frac{-1}{j\omega C} & R_2 + \frac{1}{j\omega C} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} A/\sqrt{2} \\ 0 \end{bmatrix}$$

invirtiendo la matriz:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} R_2 + \frac{1}{j\omega C} & \frac{1}{j\omega C} \\ \frac{1}{j\omega C} & R_1 + \frac{1}{j\omega C} \end{bmatrix} \begin{bmatrix} A/\sqrt{2} \\ 0 \end{bmatrix}}{\left(R_1 + \frac{1}{j\omega C}\right)\left(R_2 + \frac{1}{j\omega C}\right) - \left(\frac{1}{j\omega C}\right)^2}$$

$$\Rightarrow \quad I_1 = \frac{\frac{A}{\sqrt{2}}(1 + j\omega R_2 C)}{(R_1 + R_2) + j\omega C R_1 R_2}$$

(a)
$$I_2 = \frac{\frac{A}{\sqrt{2}}}{(R_1 + R_2) + j\omega C R_1 R_2} \quad \Rightarrow \quad i_2(t) = \frac{A\cos(\omega t + \theta)}{\sqrt{(R_1+R_2)^2 + (\omega C R_1 R_2)^2}}$$

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en que $\theta = -\text{Arctg}\left(\frac{\omega C R_1 R_2}{R_1 + R_2}\right)$

(b) $\underline{P} = \underline{V}_f \, \underline{I}_1^*$

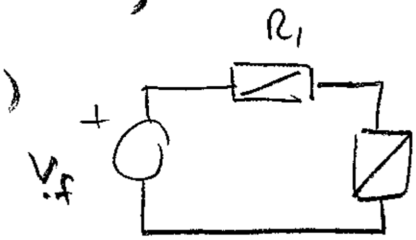
$$= \frac{A}{\sqrt{2}} \left(\frac{A}{\sqrt{2}} \, \frac{(1+j\omega R_2 C)}{(R_1+R_2)+j\omega C R_1 R_2} \right)^*$$

$$= \frac{A}{2} \, \frac{(1-j\omega R_2 C)\left[(R_1+R_2)+j\omega C R_1 R_2\right]}{(R_1+R_2)^2+(\omega C R_1 R_2)^2}$$

$$P_{ac} = \mathrm{Re}\{\underline{P}\} = \frac{A\left(R_1+R_2+\omega^2 R_1 R_2^2 C^2\right)}{2\left((R_1+R_2)^2+(\omega C R_1 R_2)^2\right)}$$

O bien,

(a)



$$\frac{1}{j\omega C} /\!/ R_2 \uparrow \underline{V}_2 = \underline{V}_f \, \frac{\frac{1}{j\omega C} /\!/ R_2}{R_1 + \frac{1}{j\omega C} /\!/ R_2}$$

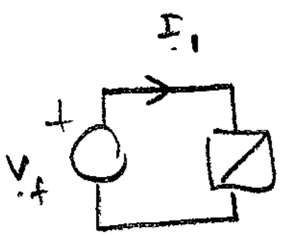
$$\frac{\frac{1}{j\omega C} \cdot R_2}{\frac{1}{j\omega C}+R_2} = \frac{R_2}{1+j\omega C R_2}$$

$$\frac{R_2}{R_1(1+j\omega C R_2)+R_2}$$

$$\Rightarrow \underline{I}_2 = \frac{\underline{V}_2}{R_2} = \frac{\underline{V}_f}{R_2} \, \frac{\frac{1}{j\omega C} /\!/ R_2}{R_1 + \frac{1}{j\omega C} /\!/ R_2}$$

$i_2(t) = \dots$ (lo mismo que antes)

(b)



$$\underline{Z}_{eq} = R_1 + \frac{1}{j\omega C} /\!/ R_2$$

2 $\boxed{\underline{P} = \underline{V}_f \, \underline{I}_1^*} = |\underline{I}_1|^2 \, \underline{Z}_{eq} = \dots$ (lo mismo que antes)

donde $\underline{I}_1 = \dfrac{\underline{V}}{\underline{Z}_{eq}}$