ELO102 – S1 2014 – Control #2 – 24 de marzo de 2014

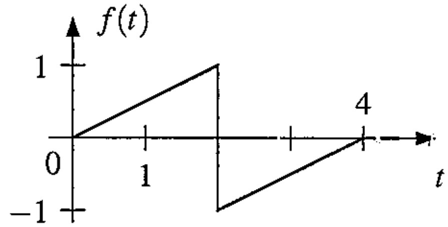
Responda SOLO UNO de los dos problemas propuestos.
Indique claramente cuál de los dos responde.

Problema 2.1 *Para la señal de la figura (suponiendo $f(t) = 0$, para $t \notin [0,4]$):*

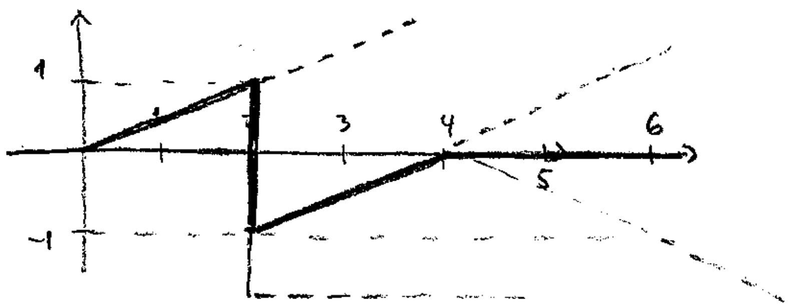
(a) Encuentre una expresión analítica,

(b) Determine y grafique su derivada y su integral definida, y

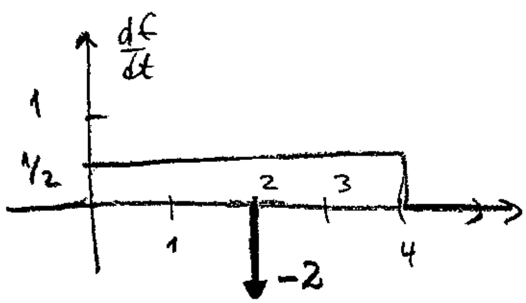
(c) Determine su valor medio y su valor efectivo en el intervalo $[0,4]$.



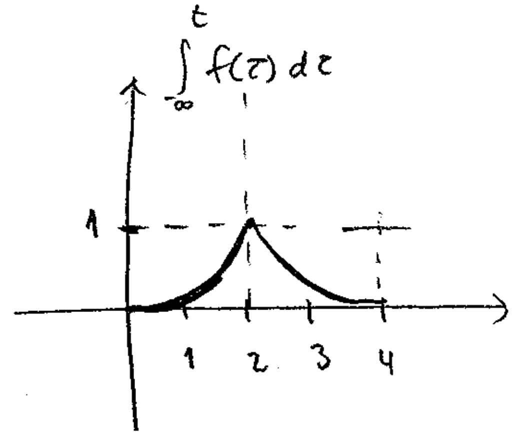
(a) $f(t) = \frac{1}{2} r(t) - 2\mu(t-2) - \frac{1}{2} r(t-4)$



(b) $\frac{df}{dt} = \frac{1}{2}\mu(t) - 2\,\delta(t-2) - \frac{1}{2}\mu(t-4)$



$$\int_{-\infty}^{t} f(\tau)\,d\tau = \begin{cases} 0 & t \leq 0 \\ \frac{1}{4}t^2 & 0 \leq t \leq 2 \\ \frac{1}{4}(t-4)^2 & 2 \leq t \leq 4 \\ 0 & t \geq 4 \end{cases}$$



Queremos la integral definida de

$$f(t) = \frac{1}{2} r(t) - 2\mu(t-2) - \frac{1}{2} r(t-4)$$

Sea $\quad p(t) = \int_{-\infty}^{t} r(\tau)\, d\tau = \frac{1}{2} t^2 \mu(t)$

$$I(t) = \int_{-\infty}^{t} f(\tau)\, d\tau = \frac{1}{4} t^2 \mu(t) - 2\, r(t-2) - \frac{1}{2} \cdot \frac{1}{2} (t-4)^2 \mu(t-4)$$

$$t \leq 0 \Rightarrow \quad I(t) = 0$$

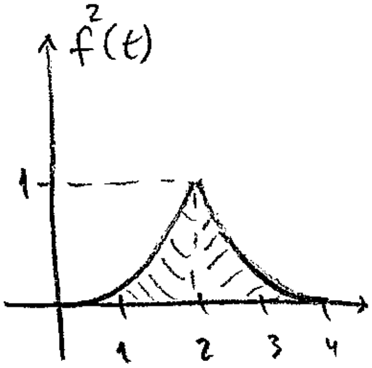
$$0 \leq t \leq 2 \Rightarrow \quad I(t) = \frac{1}{4} t^2$$

$$2 \leq t \leq 4 \Rightarrow \quad I(t) = \frac{1}{4} t^2 - 2(t-2) = \frac{1}{4} t^2 - 2t + 4 = \frac{1}{4}(t-4)^2$$

$$4 \leq t \Rightarrow \quad I(t) = \frac{1}{4} t^2 - 2(t-2) - \frac{1}{4}(t-4)^2 = 0 \quad //$$

(c) $\quad \bar{f} = \int_0^4 f(t)\, dt = 0 \quad$ (por simetría)

$$f_{rms} = \sqrt{\frac{1}{4} \int_0^4 f^2(\tau)\, d\tau} = \sqrt{\frac{1}{4} \cdot 2 \cdot \frac{1}{3}\, 2 \cdot 1} = \frac{1}{\sqrt{3}}$$



Problema 2.2 *Considere el sistema definido por*

$$r(t) = T\left\langle x(t_o) = x_o\,;\, e(t)\right\rangle = x_o + (t-t_o)\int_{t_o}^{t} e(\tau)d\tau \qquad \forall t \geq t_o$$

(a) *Determine si el sistema es lineal.*

(b) *Determine si el sistema es invariante en el tiempo.*

(a) $T\langle \alpha_1 x_{o1} + \alpha_2 x_{o2}\,;\, \beta_1 e_1(t) + \beta_2 e_2(t)\rangle$

$$= \alpha_1 x_{o1} + \alpha_2 x_{o2} + (t-t_o)\int_{t_o}^{t} \left(\beta_1 e_1(\tau) + \beta_2 e_2(\tau)\right) d\tau$$

$$= \alpha_1 x_{o1} + \alpha_2 x_{o2} + \beta_1 (t-t_o)\int_{t_o}^{t} e_1(\tau) d\tau + \beta_2 (t-t_o)\int_{t_o}^{t} e_2(\tau) d\tau$$

$$= \alpha_1 T\langle x_{o1}; 0\rangle + \alpha_2 T\langle x_{o2}; 0\rangle + \beta_1 T\langle 0; e_1(t)\rangle + \beta_2 T\langle 0; e_2(t)\rangle$$

(b) Nos interesa la respuesta del sistema cuando $\tilde{e}(t) = e(t-T)$ y $x(\underbrace{t_o+T}_{\tilde{t}_o}) = x_o$

$$\Rightarrow \tilde{r}(t) = T\langle x(t_o+T) = x_o\,;\, e(t-T)\rangle \qquad \forall t \geq t_o + T$$

$$= x_o + (t-\tilde{t}_o)\int_{\tilde{t}_o}^{t} e(\tau - T) d\tau \qquad \tilde{\tau} = \tau - T$$

$$= x_o + ((t-T)-t_o)\int_{t_o}^{t-T} e(\tilde{\tau})\, d\tilde{\tau}$$

$$= r(t-T)$$

Por tanto el sistema ES invariante en el tiempo.