Nombre: Solución

ELO102 – S1 2015 – Control #2 – 23 de marzo de 2015

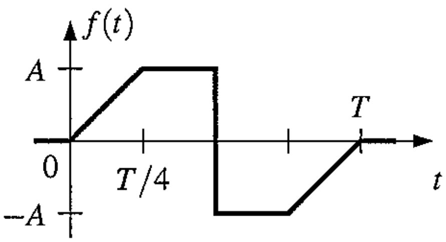
Responda SOLO UNO de los dos problemas propuestos. Indique claramente cuál responde.

Problema 2.1 *Considere la señal de la figura, suponiendo* $f(t) = 0$, *para* $t \notin [0, T]$.

(a) *Encuentre una expresión analítica para* $f(t)$, $\forall t \in \mathbb{R}$.

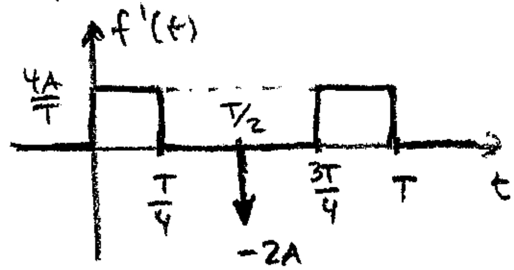
(b) *Determine y grafique la derivada de* $f(t)$.

(c) *Determine el valor efectivo o* RMS *de* $f(t)$ *en el intervalo* $[0, T]$.



(a) $$f(t) = \frac{A}{T/4} r(t) - \frac{A}{T/4} r(t - T/4) - 2A\mu(t - \tfrac{T}{2}) + \frac{A}{T/4} r(t - \tfrac{3T}{4}) - \frac{A}{T/4} r(t - T)$$

(b) $$f'(t) = \frac{4A}{T}\mu(t) - \frac{4A}{T}\mu(t - T/4) - 2A\delta(t - T/2) + \frac{4A}{T}\mu(t - \tfrac{3T}{4}) - \frac{4A}{T}\mu(t - T)$$



(c) $$f_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)\,dt = \frac{2}{T}\int_0^{T/2} f^2(t)\,dt = \frac{2}{T}\left[\int_0^{T/4}\left(\frac{4A}{T}t\right)^2 dt + \int_{T/4}^{T/2} (A)^2\, dt\right]$$

$$= \frac{2}{T}\left[\frac{16A^2}{T^2}\,\frac{1}{3}\left(\frac{T}{4}\right)^3 + A^2\,\frac{T}{4}\right]$$

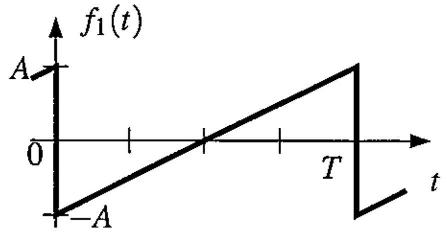
$$= A^2\left[\frac{1}{6} + \frac{1}{2}\right] = \frac{2}{3}A^2$$

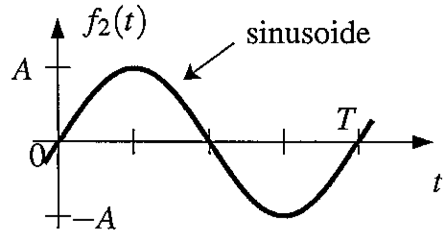
$$\Rightarrow f_{rms} = \sqrt{\frac{2}{3}}\,A$$

Problema 2.2 *Las señales de la figura tienen el mismo período T, la misma amplitud A y valor medio igual a cero.*

(a) *Determine y grafique la derivada de cada una de ellas.*

(b) *Determine cuál tiene mayor valor efectivo o* RMS.



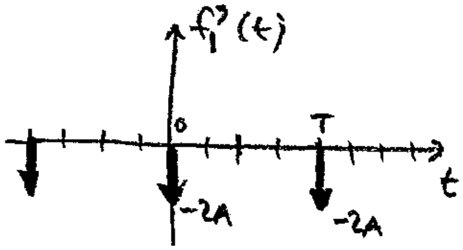


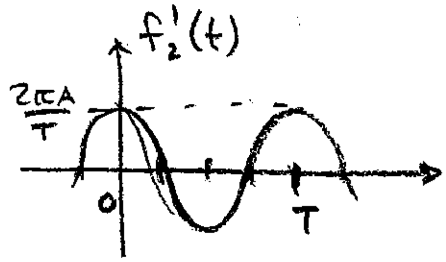
(a) $f_1(t) = \ldots + \frac{2A}{T} r(t+\frac{T}{2}) - 2A\mu(t) - \frac{2A}{T} r(t-\frac{T}{2}) + \ldots$

$f_2(t) = A \operatorname{sen}(\frac{2\pi}{T} t)$

$\Rightarrow f_1'(t) = \ldots + \frac{2A}{T}\mu(t+\frac{T}{2}) - 2A\delta(t) - \frac{2A}{T}\mu(t-T/2) + \ldots$

$f_2'(t) = \frac{2\pi}{T} A \cos(\frac{2\pi}{T} t)$





(b) $(f_1)^2_{RMS} = \frac{1}{T}\int_0^T f_1^2\, dt = \frac{1}{T}\int_0^T \left(-A + \frac{2A}{T}t\right)^2 dt = \frac{1}{T}\left[A^2 T - \frac{4A^2}{T}\frac{1}{2}T^2 + \frac{4A^2}{T^2}\frac{1}{3}T^3\right]$

$= A^2\left[1 - 2 + \frac{4}{3}\right] = \frac{1}{3}A^2$

$\Rightarrow (f_1)_{RMS} = \frac{1}{\sqrt{3}}A$

$(f_2)^2_{RMS} = \frac{1}{T}\int_0^T f_2^2\, dt = \frac{1}{T}\int_0^T A^2 \operatorname{sen}^2(\frac{2\pi}{T}t)\, dt$

$= \frac{A^2}{T}\int_0^T \frac{1}{2}\left(-\cos(\frac{4\pi t}{T}) + 1\right) dt \quad \Rightarrow (f_2)_{RMS} = \frac{A}{\sqrt{2}} > (f_1)_{RMS}$

$= 0 + \frac{A^2}{2}$