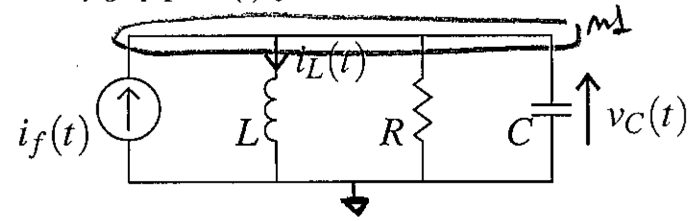
Nombre: Solución

ELO102 – S1 2017 – Control #9 – 2 de junio de 2017

Responda SOLO UNO de los dos problemas propuestos. Indique claramente cuál responde.

Problema 9.1 *En la red de la figura, $R = 1[k\Omega]$, $C = 1[\mu F]$. $L = 1[H]$, $v_C(0) = 0[V]$, $i_L(0) = 0[mA]$ e $i_f(t) = 10\mu(t)[mA]$. Determine y grafique $i_L(t)$, para $t \geq 0$.*



Se puede hacer LCK en nodo 1:

$$i_f = i_L(t) + i_R(t) + i_C(t)$$

$$i_f(t) = i_L(t) + \frac{1}{R} v_R(t) + C\frac{dv_C}{dt} \qquad \text{pero } v_R = v_C = v_L = L\frac{di_L}{dt}$$

$$\boxed{i_f(t) = i_L(t) + \frac{L}{R}\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2}}$$

Reemplazando valores

$$10\mu(t) = i_L + \frac{di_L}{dt} + \frac{d^2 i_L}{dt^2} \qquad \text{pues todas las unidades son consistentes con } t \text{ en } [ms]$$

$$i_L(t) = \underbrace{i_{L,h}(t)}_{\substack{\text{solución}\\ \text{homogénea}\\ e^{\lambda t}}} + \underbrace{i_{L,p}(t)}_{\substack{\text{solución}\\ \text{particular}\\ K}}$$

Solución homogénea:

$$i_{L,h}(t) = e^{\lambda t}$$

$$\Rightarrow \lambda^2 + \lambda + 1 = 0$$

$$\lambda_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

oscilaciones amortiguadas

Solución particular:

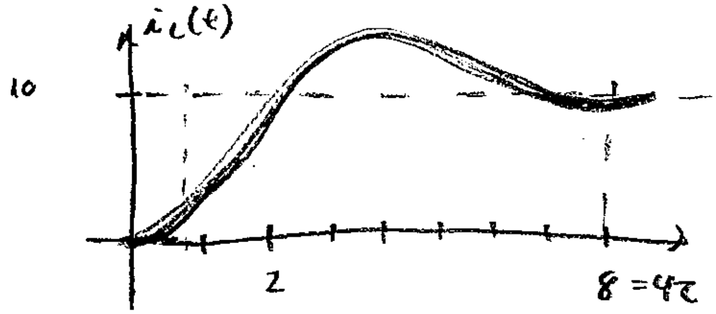
en la EDO $\quad 10 = K + \cancel{\frac{d}{dt}(K)}^{0} + \cancel{\frac{d^2}{dt^2}(K)}^{0}$

$$K = 10$$

$$\therefore i_L(t) = C_1 e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right) + C_2 e^{-\frac{t}{2}}\operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right) + 10$$

y debe satisfacer c.i. $\begin{cases} i_L(0) = 0 \Rightarrow C_1 = -10 \\ v_C(0) = 0 \Rightarrow \left.\frac{di_L}{dt}\right|_{t=0} = 0 \Rightarrow -\frac{C_1}{2} + \frac{C_2\sqrt{3}}{2} = 0 \end{cases}$

$$\Rightarrow C_2 = \frac{C_1}{\sqrt{3}} = -\frac{10}{\sqrt{3}}$$

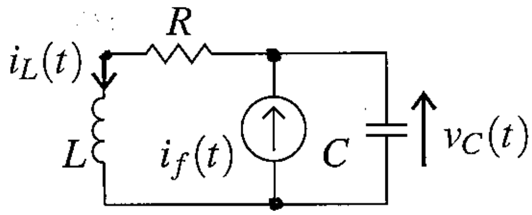


$$\omega = \frac{\sqrt{3}}{2} \approx 0.85$$

$$T = \frac{2\pi}{\omega} \approx \frac{6.3}{0.85} \approx 7{,}5$$

Solución

Problema 9.2 *En la red de la figura, $R = 0{,}5[k\Omega]$, $C = 1[\mu F]$. $L = 1[H]$, $v_C(0) = 0[V]$, $i_L(0) = 0[mA]$ e $i_f(t) = 100\mu(t)[mA]$. Determine y grafique $v_C(t)$, para $t \geq 0$.*



LCK en cada nodo: $\frac{1}{L} D^{-1} v_L + \frac{(v_L - v_c)}{R} = 0$

$\frac{(v_c - v_L)}{R} + C \frac{dv_c}{dt} = i_f(t)$

De la primera $v_L = \frac{v_c}{\frac{R}{L} D^{-1} + 1}$

en la segunda $\frac{1}{R} v_c - \frac{1}{R} \left(\frac{v_c}{\frac{R}{L} D^{-1} + 1} \right) + C D v_c = i_f(t)$

$$\frac{1}{R} \left(\frac{R}{L} D^{-1} + 1 \right) v_c - \frac{1}{R} v_c + C \left(\frac{R}{L} D^{-1} + 1 \right) D v_c = \left(\frac{R}{L} D^{-1} + 1 \right) i_f$$

$$\frac{1}{L} D^{-1} v_c + \frac{RC}{L} v_c + C D v_c = \frac{R}{L} D^{-1} i_f + i_f$$

$$\Rightarrow \quad C D^2 v_c + \frac{RC}{L} D v_c + \frac{1}{L} v_c = D i_f + \frac{R}{L} i_f$$

$$\Rightarrow \quad LC \frac{d^2}{dt^2} v_c + RC \frac{d}{dt} v_c + v_c = L \frac{d}{dt} i_f + R i_f$$

Reemplazando, $\underbrace{\frac{d^2}{dt^2} v_c + \frac{1}{2} \frac{dv_c}{dt} + v_c}_{\lambda^2 + \frac{1}{2}\lambda + 1 = 0} = \frac{d}{dt} i_f + \frac{1}{2} i_f$

Solución homogénea: $\lambda_{1,2} = -\frac{1}{4} \pm j \frac{1}{4} \sqrt{15} \Rightarrow v_{c,h}(t) = e^{-t/4} \left(A \cos\left(\frac{\sqrt{15}}{4} t \right) + B \operatorname{sen}\left(\frac{\sqrt{15}}{4} t \right) \right)$

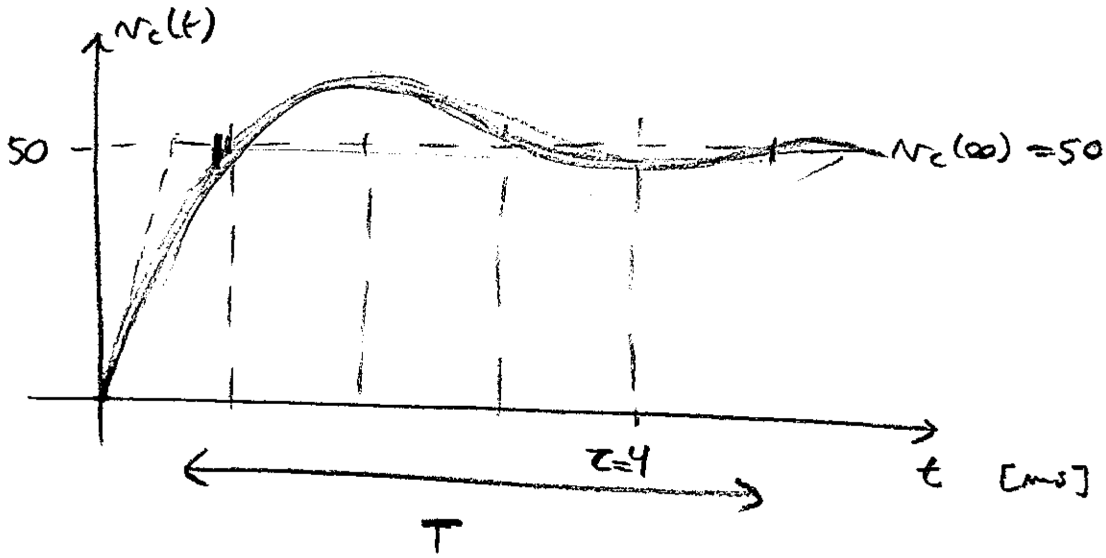
Solución particular $v_{c,p}(t) = k \Rightarrow$ en la EDO

$$\frac{d^2}{dt^2}(k) + \frac{1}{2} \frac{d}{dt}(k) + k = \frac{d}{dt}(100) + \frac{1}{2}(100) \Rightarrow k = 50\,[V]$$

Las condiciones iniciales son

$$v_c(0) = 0$$

$$\left.\frac{dv_c}{dt}\right|_{t=0} = \frac{1}{C}\left(i_f(0) - i_L(0)\right) = 100 \ \left[\frac{V}{ms}\right]$$



$$\omega = \frac{\sqrt{15}}{4} \approx 1 \Rightarrow T \approx \frac{2\pi}{\omega} \approx 6{,}3$$